A Physarum-inspired Algorithm for Minimum-cost Relay Node Placement in Wireless Sensor Networks: the supplement

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Roadmap of this supplement: In Section I, we provide the proofs for the time complexities of our PSTA and Guha and Khuller's $1.6103ln|T|$ approximation algorithm [1]; in Section II, we conduct computational trials to show the solution quality of our PSTA by varying K; and ultimately, in Section III, we evaluate the designed wireless sensor networks in our paper based on a widely-used shortest distance routing strategy (e.g. [2]).

I. THE PROOFS FOR THE TIME COMPLEXITIES

In this section, we provide the rigorous proofs for the time complexities of PSTA and Guha and Khuller's $1.6103ln|T|$ approximation algorithm [1], i.e., GKA in our paper.

Theorem 7. *PSTA has a time complexity of $O(K|E| + K|V|log|V|)$.*

Proof. First, there are two outer-loop steps: updating edge costs (Step 1; $O(|E|)$) and finding the Minimum Spanning Tree (MST) (Step 2; $O(|E| + |V|log|V|)$; Prim's algorithm [3]). Then, there are K loops (Step 3). In each loop, we associate nodes with random pressure values (Step 4; $O(|V|)$), and calculate fluxes using these pressures (Step 5; $O(|E|)$). Subsequently, we construct $CCG'(V, E, c')$ and use the MST technique to produce a feasible solution (Steps 6-11; $O(|E| + |V|log|V|)$). Clearly, the time complexity of K loops is $O(K|E| + K|V|log|V|)$, which is larger than those of two outer-loop steps. Hence, this theorem holds. □

Theorem 8. *Guha and Khuller's $1.6103ln|T|$ approximation algorithm [1] has a time complexity of $O(|T||V||E| + |T||V|^2log|V|)$.*

Proof. First, a spider with the minimum ratio needs to be found using Klein and Ravi's method [5], which involves finding the lowest-cost path, where the sum of node weights and edge costs is minimized, between each pair of vertices. Given that the lowest-cost path can be found by embedding
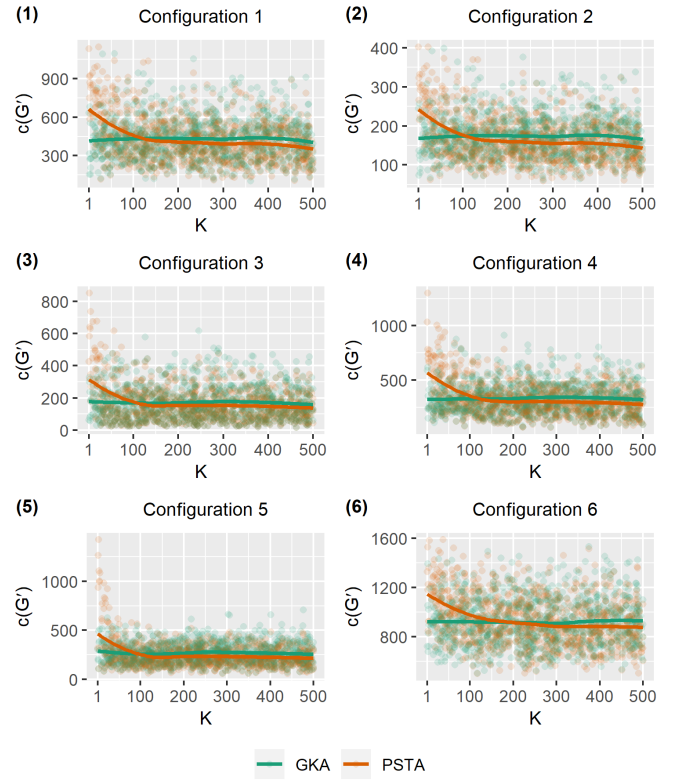
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Figure 1: The solution quality of PSTA: varying K. The dots are the solutions of PSTA and GKA. The lines are the Locally Estimated Scatterplot Smoothing lines [4]. The parameter K is in PSTA, but not in GKA. We visualize the solutions of GKA to the same K with the solutions of PSTA for each instance. Configuration 1: $|B| = 2$, $|S| = 100$, $|R| = 20$, $\alpha = 150$, $\gamma_{th} = 6E-10$. Configuration 2 is different from Configuration 1 in that $\alpha = 50$. Configuration 3 is different from Configuration 1 in that $|B| = 8$. Configuration 4 is different from Configuration 1 in that $|S| = 90$. Configuration 5 is different from Configuration 1 in that $|R| = 35$. Configuration 6 is different from Configuration 1 in that $\gamma_{th} = 12E-10$.

node weights onto edges and then finding the shortest path [6] ($O(|E| + |V|log|V|)$; Dijkstra's algorithm [7]) , the time complexity of finding this spider is $O(|V||E| + |V|^2log|V|)$. In scenarios where this spider is not a 3+ spider, a second spider and a forest need to be found, which involves finding

the lowest-cost path between each pair of terminal trees ($O(|V||E| + |V|^2 log|V|)$). We use a spider or a forest to concatenate terminal trees. The time complexity of such a concatenation is $O(|V||E| + |V|^2 log|V|)$. The maximum number of such concatenations is $|T| - 1$, i.e., two terminal trees are concatenated each time. Hence, this theorem holds. □

II. THE SOLUTION QUALITY OF PSTA: VARYING K

In this section, we conduct computational trials to show the solution quality of PSTA by varying K.

Visualizing the computational results: We visualize the computational results in Figure 1 in this supplement. In each subfigure, e.g. Figure 1(1), we fix a configuration of five parameters ($|B|$, $|S|$, $|R|$, α, γ_{th}), and randomly generate 1000 instances in the same way as that in our paper. We apply PSTA and GKA, i.e., Guha and Khuller's $1.6103 ln|T|$ approximation algorithm [1], to solve each of these instances. To apply PSTA to solve each of these instances, we randomly select the value of K between 1 and 500. We use scatterplot to visualize the solutions of PSTA and GKA. Since there is no parameter K in GKA, we visualize the solutions of GKA to the same K with the solutions of PSTA for each instance. The purpose of doing this is to visualize the comparison between the solutions of PSTA and GKA clearly. Moreover, we use the Locally Estimated Scatterplot Smoothing lines [4] to compare the solutions of PSTA and GKA statistically.

Evaluating the computational results: In Figure 1 in this supplement, we apply six configurations of five parameters ($|B|$, $|S|$, $|R|$, α, γ_{th}) for generating different instances. Configuration 1 is the base configuration, and Configurations 2-6 are used for varying five parameters separately. For every configuration, PSTA generally produces worse solutions than GKA when K is smaller than 100, and better solutions than GKA when K is larger than 300. These computational results show that whether PSTA is likely to perform better or worse than GKA mainly depends on K, and PSTA can generally produce better solutions than GKA when K is large.

III. EVALUATING WIRELESS SENSOR NETWORKS: THE SHORTEST DISTANCE ROUTING STRATEGY

In this section, we evaluate the designed wireless sensor networks in our paper based on a widely-used shortest distance routing strategy, i.e., sensor nodes transmit data to base stations via paths in which the sums of distances of transmission routes are minimized (e.g. [2]). We visualize the evaluation results in Figure 2 in this supplement. The only differences between Figure 2 in this supplement and Figure 6 in our paper are values of t_{net}, t_{delay}, and $g_\%$. In Figure 2 in this supplement, these values are calculated based on the shortest distance routing strategy, while in Figure 6 in our paper, these values are calculated based on the minimum-sum-of-outage-probability routing strategy, i.e., sensor nodes transmit data to base stations via paths in which the sums of outage probabilities of transmission routes are minimized.

Varying α: We vary α in Figure 2a in this supplement. Like the computational results in our paper, when $\alpha = 1$, the WSNs designed by PSTA and GKA generally have larger t_{net}, smaller t_{delay}, and larger $g_\%$ than those designed by OSRP and

RRPL, while when $\alpha = 150$, the WSNs designed by PSTA and GKA generally have smaller t_{net}, larger t_{delay}, and smaller $g_\%$ than those designed by OSRP and RRPL. The reason is that a small α weights the outage probabilities of transmission routes more than the costs of relay nodes.

Varying $|B|$: We vary $|B|$ in Figure 2b in this supplement. Like the computational results in our paper, when $|B|$ is large, the designed WSNs have larger t_{net}, smaller t_{delay}, and larger $g_\%$. The reason is that fewer relay nodes are in the middle of sensor nodes and base stations when $|B|$ is large.

Varying $|S|$: We vary $|S|$ in Figure 2c in this supplement. Like the computational results in our paper, when $|S|$ is large, the designed WSNs generally have smaller t_{net}, larger t_{delay}, and smaller $g_\%$. The reason is that, when $|S|$ is large, the generated instances are sparser, and more relay nodes are in the middle of sensor nodes and base stations.

Varying $|R|$: We vary $|R|$ in Figure 2d in this supplement. Like the computational results in our paper, when $|R|$ is large, the designed WSNs generally have smaller t_{delay}. Different from the computational results in our paper, t_{net} and $g_\%$ do not increase much with $|R|$. The reason is that, even though a large $|R|$ may provides transmission routes with smaller outage probabilities, the shortest distance routing strategy may not employ these transmission routes.

Varying γ_{th}: We vary γ_{th} in Figure 2e in this supplement. Like the computational results in our paper, when γ_{th} is small, the designed WSNs have larger t_{net}, smaller t_{delay}, and larger $g_\%$. There are two reasons for this: 1) a small γ_{th} induces large transmission ranges of devices, and as a result, a large number of transmission routes; and 2) a small γ_{th} induces small outage probabilities of transmission routes (details in [8]).

Varying M_r: We vary M_r in Figure 2f in this supplement. Like the computational results in our paper, t_{delay} and $g_\%$ increase with M_r. The reason is that a large M_r induces large success probabilities of transmissions between devices, at the cost of long delays (details in [8]).

The usefulness of our PSTA: Like the computational results in our paper, PSTA can design cheap WSNs with a high QoS. For example, in comparison to RRPL, our PSTA can design WSNs with 25% lower relay cost and similar qualities of service by average (specifically, 3% shorter network lifetime, 4% longer delay, and 0% loss of goodput) when $|B| = 2$, $|S| = 100$, $|R| = 20$, $\alpha = 150$, $\gamma_{th} = 6E-10$, $M_r = 100$ (details in Table I in this supplement). Therefore, the major conclusion in our paper, i.e., in comparison to two state-of-the-art relay node placement algorithms, our PSTA can design cheaper WSNs with similar quality of service, also holds for the shortest distance routing strategy.

Table I: The comparison when $|B| = 2$, $|S| = 100$, $|R| = 20$, $\alpha = 150$, $\gamma_{th} = 6E-10$, $M_r = 100$ (base algorithm: RRPL).

Algorithm	t_{net}	t_{delay}	$g_\%$	$cost_{relay}$	t_{run}
PSTA	0.97*base	1.04*base	1.00*base	0.75*base	74.32*base
GKA	0.98*base	1.05*base	1.00*base	0.84*base	2949*base
OSRP	0.97*base	1.02*base	1.00*base	1.22*base	16.13*base
RRPL	1.00*base	1.00*base	1.00*base	1.00*base	1.00*base

(a) Varying α ($|B| = 2$, $|S| = 100$, $|R| = 20$, $\gamma_{th} = 6E{-}10$, $M_r = 10$)

(b) Varying $|B|$ ($|S| = 100$, $|R| = 20$, $\alpha = 150$, $\gamma_{th} = 6E{-}10$, $M_r = 10$)

(c) Varying $|S|$ ($|B| = 2$, $|R| = 20$, $\alpha = 150$, $\gamma_{th} = 6E{-}10$, $M_r = 10$)

(d) Varying $|R|$ ($|B| = 2$, $|S| = 100$, $\alpha = 150$, $\gamma_{th} = 6E{-}10$, $M_r = 10$)

(e) Varying γ_{th} ($|B| = 2$, $|S| = 100$, $|R| = 20$, $\alpha = 150$, $M_r = 10$)

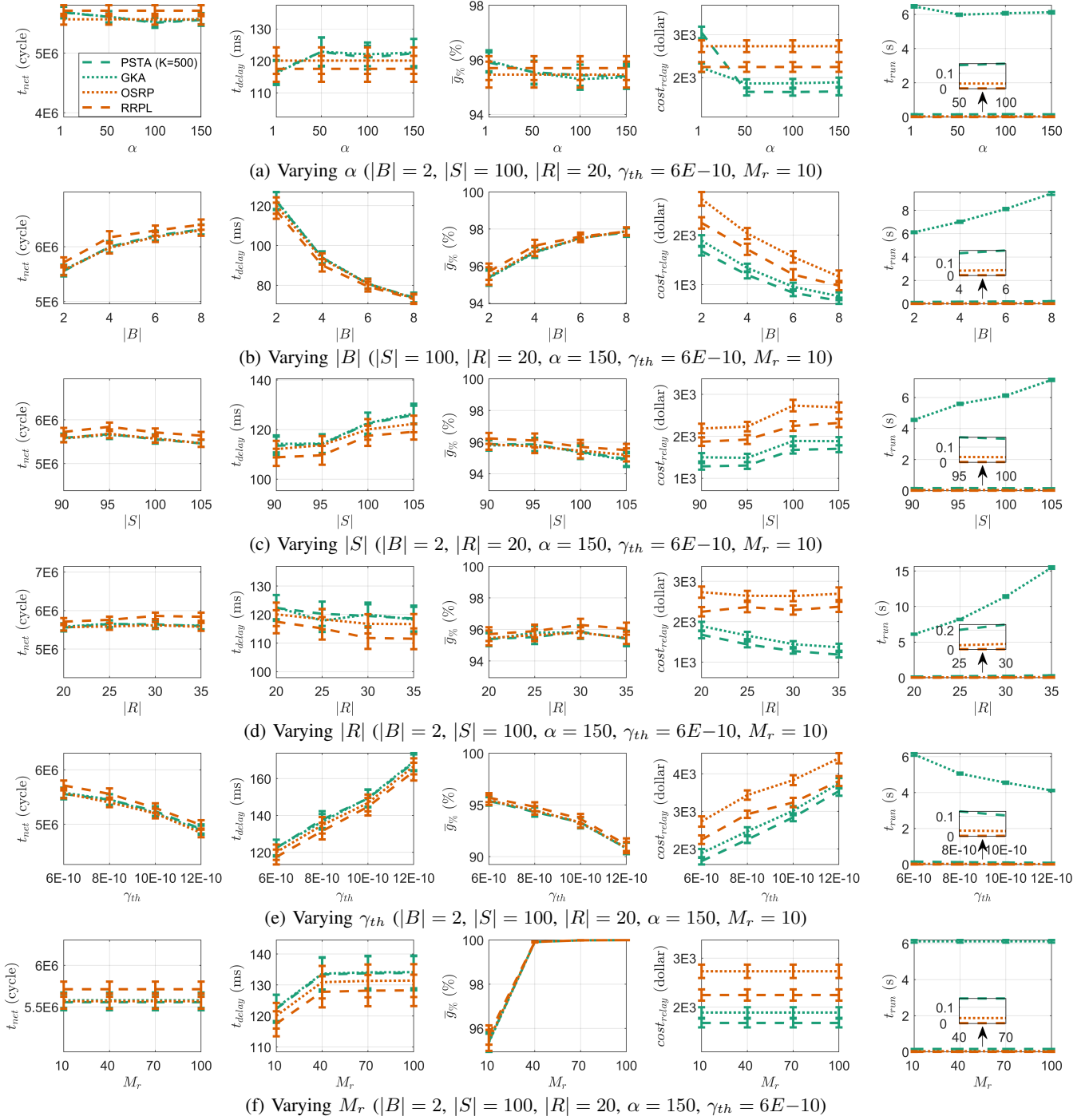(f) Varying M_r ($|B| = 2$, $|S| = 100$, $|R| = 20$, $\alpha = 150$, $\gamma_{th} = 6E{-}10$)

Figure 2: Evaluating the designed wireless sensor networks based on the shortest distance routing strategy.

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