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On-orbit Calibration of Star Sensor Based on a New Lens Distortion Model

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Abstract: A new lens distortion model is built to eliminate the adverse effect of lens distortion on star sensors, and an on-orbit calibration algorithm which is based on the new model is proposed. In view of the conventional lens distortion model's defects, that are the conventional model doesn't tolerate the variation of temperature and it's based on the over-simplified pin-hole camera model, the new lens distortion model is built by simulating and analyzing the image data of a wide-field and large-aperture star sensor lens. There are several distortion coefficients in this new distortion model, which include the thermal coefficients that can be used to describe the effect of thermal environment's change on lens distortion. Afterward an on-orbit EKF algorithm is created to calibrate these distortion coefficients. And the simulation's results reveal that the precision of star sensor has been improved to 0.279" by this EKF algorithm, therefore the new lens distortion model and the new calibration algorithm are effective to eliminate the adverse effect of lens distortion on star sensors.

Key Words: Star sensor, Attitude determination, Lens distortion, Extended Kalman Filter

1 Introduction

Star sensor is the most precise attitude sensors nowadays. However, the researches on this field are inadequate in China. As a result, the precision of attitude determination by star sensor is low. As a matter of fact, star sensor is an optical system working in the space and the lens distortion caused by the abominable working environment is the main reason for its errors.

The foreign researches on lens distortion are much thorough, and the method of eliminating effects of lens distortion with a distortion polynomial presented in Ref. [1-2] is the dominating method in this field at the moment. Therefore, the Chinese researchers prefer this method to calibrate the lens distortion of star sensor, such as Ref. [3]. Nonetheless, there is a conspicuous disadvantage in this method when it's applied in the space environment. That is to say it doesn't take full account of the complexity and asperity of the space environment. The main reason is that the rapid variation of temperature will make this calibration method ineffective. Ref. [4-5] have deeply analyzed the effects of heat on star sensors, and they reveal that the variation of temperature has a big influence on star sensor's lens distortion. Moreover, the previous method is based on the over-simplified pin-hole camera model. Hence we can't get a favorable result by using this conventional method to calibrate star sensor's lens distortion.

The French SED star senor has considered the thermal effects in its design process, which is cited in Ref. [6]. However, the specific method is not published. This paper uses ZEMAX to build a star senor lens model at first and then the imaging data under different temperatures is obtained by simulating the lens model. In addition, a new lens distortion model is built and tested with the imaging data. Ultimately, an EKF algorithm is presented to achieve the on-orbit calibration of star sensor.

2 Conventional lens distortion model

The imaging principle of star sensor is presented in Fig.1, and $O_S X_S Y_S Z_S$ is the coordinate system of star sensor, α is the angle between the optical axis and the incident light, β is the angle between the optical axis and the emergent light, f is the focal length. The conventional lens distortion model is based on the pin-hole camera model, in which α is equal to β. Actually, the two angles are not equal and β will vary following the temperature even if α is a constant.

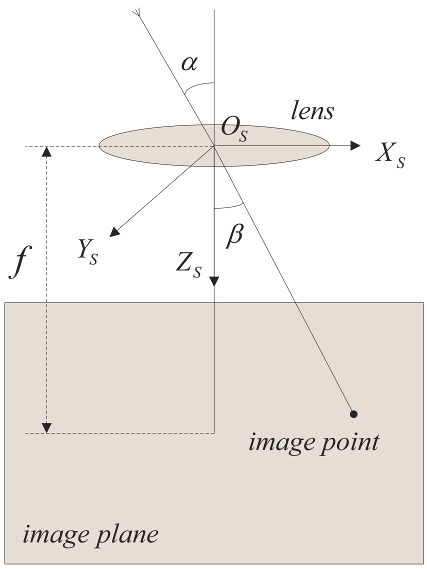

Fig. 1: Theory of star sensor imaging

There are three kinds of lens distortion in the conventional lens distortion model [7], which are radial distortion, centrifugal distortion and thin prism distortion. The method of eliminating effects of lens distortion with a distortion polynomial is the main content of the conventional model. Lens distortion can usually be expressed by equation (1).

$$\begin{cases} u_d = u + \delta_u(u,v) \\ v_d = v + \delta_v(u,v) \end{cases} \quad (1)$$

where u and v are the unobservable distortion-free coordinates; u_d and v_d are the corresponding distorted imaging coordinates. $\delta_u(u,v)$ and $\delta_v(u,v)$ are the distortion in u and v direction respectively, which can be classified into three types: radial distortion, centrifugal distortion and thin prism distortion.

Radial distortion is caused by the flawed radial curvature of lens and it's governed by equation (2).

$$\begin{cases}\delta_{ur}(u,v)=u(k_1r^2+k_2r^4)\\ \delta_{vr}(u,v)=v(k_1r^2+k_2r^4)\end{cases} \tag{2}$$

where k_1 and k_2 are radial distortion coefficients; r is the distance between the image point (u,v) and the center of image plane.

Centrifugal distortion comes from the various centrifugations of lens and it can be described by equation (3).

$$\begin{cases}\delta_{ud}(u,v)=p_1(3u^2+v^2)+2p_2uv\\ \delta_{vd}(u,v)=p_2(3v^2+u^2)+2p_1uv\end{cases} \tag{3}$$

where p_1 and p_2 are coefficients of centrifugal distortion.

Thin prism distortion arises mainly from the tilt of lens with respect to the image plane and it can be represented by equation (4).

$$\begin{cases}\delta_{up}(u,v)=s_1(u^2+v^2)\\ \delta_{vp}(u,v)=s_2(u^2+v^2)\end{cases} \tag{4}$$

where s_1 and s_2 are coefficients of thin prism distortion.

Then the general lens distortion of star sensor can be expressed as

$$\begin{cases}\delta_u(u,v)=k_1u(u^2+v^2)+k_2u(u^2+v^2)^2+\\ \quad p_1(3u^2+v^2)+2p_2uv+s_1(u^2+v^2)\\ \delta_v(u,v)=k_1v(u^2+v^2)+k_2v(u^2+v^2)^2+\\ \quad p_1(3v^2+u^2)+2p_2uv+s_1(u^2+v^2)\end{cases} \tag{5}$$

Equation (5) is based on the undistorted image coordinates and the undistorted image point (u,v) is transformed into the distorted image point (u_d,v_d). In the calibration of lens distortion an reversed equation is often used, which is described as

$$\begin{cases}u=u_d+\delta_{u_d}(u_d,v_d)\\ v=v_d+\delta_{v_d}(u_d,v_d)\end{cases} \tag{6}$$

where

$$\begin{cases}\delta_{u_d}(u_d,v_d)=k_1u_d(u_d{}^2+v_d{}^2)+k_2u_d(u_d{}^2+v_d{}^2)^2+\\ \quad p_1(3u_d{}^2+v_d{}^2)+2p_2u_dv_d+s_1(u_d{}^2+v_d{}^2)\\ \delta_{v_d}(u_d,v_d)=k_1v_d(u_d{}^2+v_d{}^2)+k_2v(u_d{}^2+v_d{}^2)^2+\\ \quad p_1(3v_d{}^2+u_d{}^2)+2p_2u_dv_d+s_1(u_d{}^2+v_d{}^2)\end{cases} \tag{7}$$

Equation (1) and (6) constitute the conventional lens distortion model, and the distortion coefficients are related to the temperature of working environment [8], therefore the degree of lens distortion varies following the temperature and we can't get an excellent calibration result with this conventional model.

3 New lens distortion model

The temperature of the space environment always vary dramatically, hence it's necessary to build a new lens distortion model which can tolerate the variation of temperature. First of all, a model of star sensor lens must be built. The optical system of star sensor has an apparent aberration and the main objective of its design is to access the known imaging model as much as possible. A model of star sensor lens with wide field and large aperture is built by using ZEMAX. The design temperature of this lens is 0°C. The lens' specific design indexes [9] are presented in Table. 1. And its 2D structure is showed in Fig. 2.

Table 1: Design Indexes of Star Sensor Lens

Structure of lens	Double Gauss
Focal length	45mm
Relative aperture	1/1.2
Field of view (FOV)	25°
Design wavelength	540nm
Material	SF1, BAF8

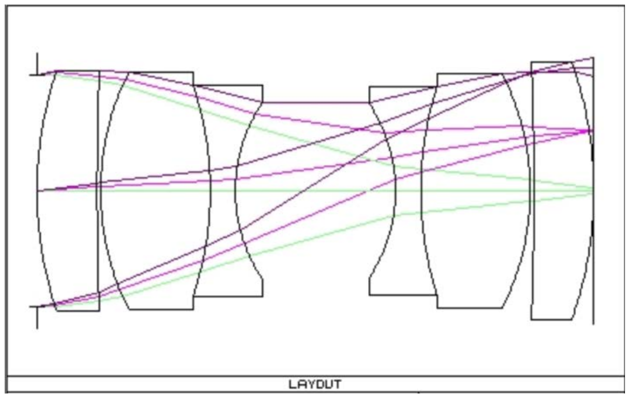

Fig. 2: 2D structure of star sensor lens model

It has been referred in the second chapter that the conventional lens distortion model is based on the over-simplified pin-hole camera model, in which α is equal to β. Virtually, the two angles are not equal. The pin-hole camera model can be applied to a star sensor with narrow field when we don't emphasize its precision very much. However, it's not applicable to star sensors with wide field, which are the main types of star sensors, and the main reason is that star sensors with wide field are extremely susceptible to the thermal variation of working environment. The distinction of wide-field star sensors' lens distortion under different temperatures is apparent. Therefore, the conventional lens distortion model has two defects when it's applied to star sensor, that are it doesn't tolerate the variation of temperature and it's based on the over-simplified pin-hole camera model. A new lens distortion model is built by simulating and analyzing imaging data of the lens model above. The new model has remedied the two kinds of errors due to the fact that it uses the numerical relationship of α and β to describe the degree of lens distortion. Δ represents the degree of lens distortion in the new model.

$$\Delta=\alpha-\beta \tag{8}$$

And Δ can be classified into two kinds: systematic distortion Δ_S and thermal distortion Δ_t. The systematic distortion is caused by the defective lens structure and the over-simplified pin-hole camera model. The thermal distortion is caused by the divergence between the actual temperature of working environment and the design temperature of star sensor lens. The two kinds of distortion can be expressed by equation (9):

$$\begin{cases} \Delta_S = f(\beta) \\ \Delta_t = f(\beta, t) \end{cases} \tag{9}$$

where t is the temperature of working environment.

Suppose $\boldsymbol{W} = (l, m, n)^{\mathrm{T}}$ is the unit direction-vector of the illuminant in star sensor coordinate system, and the position of illuminant can be determined by the X-direction field angle θ_x and the Y-direction field angle θ_y. These two angles are defined as:

$$\begin{aligned} \tan\theta_x &= \frac{l}{n}, \\ \tan\theta_y &= \frac{m}{n} \end{aligned} \tag{10}$$

With the help of ZEMAX's thermal analysis module and its function of ray tracing, the positions of image point under diverse thermal environments can be obtained by fixing several illuminants.

Next, the numerical relationship of α and β is got by analyzing the imaging data of the lens model and it has been found that there is a polynomial relationship between these two angles. Therefore the numerical relationship of α and β is described by equation (11).

$$\alpha = a_1\beta^3 + a_2\beta^2 + a_3\beta + a_4t^3 + a_5t^2 + a_6t + a_7 \tag{11}$$

where a_1 to a_7 are the distortion coefficients. We can acquire them with the fitting method or the estimation method.

The root mean square error (RMSE) of α and β is 0.028 before the lens distortion is calibrated by the new model. On the contrary, the new model can decrease their RMSE to 0.0015, which reveals that the new lens distortion model is effective. And Fig. 3 further demonstrates the effectiveness of this new model to calibrate the lens distortion of star sensor.

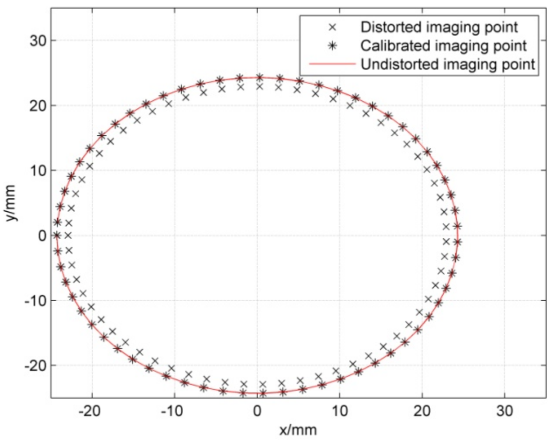

Fig. 3: Calibration effect of new lens distortion model

4 On-orbit calibration of star sensor

4.1 Attitude determination based on the new model

Star sensor is an optical system working in the space, and it determines the satellite's attitude by imaging target stars. Suppose the main point drift and the variation of focal length have been calibrated, and $\boldsymbol{W} = (l, m, n)^{\mathrm{T}}$ is the unit direction-vector of the target star in star sensor coordinate system. The process of attitude determination of star sensor based on the new lens distortion model can be described as follows:

1) The positions of image points (x, y, f) are obtained by imaging target stars, and then β can be calculated by equation (12).

$$\beta = \arccos\frac{f}{\sqrt{x^2 + y^2 + f^2}} \tag{12}$$

2) Then α is got by using equation (11).

3) The present productive technology guarantees the axial symmetry of lens. Moreover, the incident light, emergent light and the optical axis are in a plane if the lens is axial symmetric. And the unit direction-vector of the target stars in star sensor coordinate system can be obtained based on this fact. It's expressed by equation (13):

$$\boldsymbol{W} = (l, m, n)^{\mathrm{T}} = \begin{bmatrix} -x\sin\alpha\sqrt{\dfrac{1}{x^2+y^2}} \\ -y\sin\alpha\sqrt{\dfrac{1}{x^2+y^2}} \\ -\cos\alpha \end{bmatrix} \tag{13}$$

4) The target stars can be distinguished by star-map matching. And the positions of target stars are constant in the inertial coordinate system, therefore their unit direction-vectors in the inertial coordinate system V can be described with stars' right ascension α_i and declination δ_i

$$\boldsymbol{V} = \begin{bmatrix} \cos\alpha_i \cos\delta_i \\ \sin\alpha_i \cos\delta_i \\ \sin\delta_i \end{bmatrix} \tag{14}$$

5) Suppose the number of target stars in a photo is n, and the unit direction-vector of the Num. i target star in star sensor coordinate system is $\boldsymbol{W}_i$, the unit direction-vector of the Num. i target star in inertial coordinate system is $\boldsymbol{V}_i$. Next, the satellite's attitude quaternion can be attained with the algorithm of QUEST. In QUEST, a matrix K is described by equation (15).

$$\mathrm{K} = \begin{bmatrix} S - \sigma I & Z \\ Z^{\mathrm{T}} & \sigma \end{bmatrix} \tag{15}$$

where

$$\begin{aligned} \sigma &= tr\left(\sum_{i=1}^{n}\frac{1}{n}\boldsymbol{W}_i\boldsymbol{V}_i^{\mathrm{T}}\right), \\ S &= \sum_{i=1}^{n}\frac{1}{n}\left(\boldsymbol{W}_i\boldsymbol{V}_i^{\mathrm{T}} + \boldsymbol{V}_i\boldsymbol{W}_i^{\mathrm{T}}\right), \\ Z &= \sum_{i=1}^{n}\frac{1}{n}\left(\boldsymbol{W}_i * \boldsymbol{V}_i\right) \end{aligned} \tag{16}$$

The satellite's attitude quaternion Q is the eigenvector corresponded by the maximal characteristic value of K.

$$Q=\begin{bmatrix} q_0 \\ \boldsymbol{q} \end{bmatrix}=\begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^{\mathrm{T}} \tag{17}$$

6) Transform the attitude quaternion Q to attitude matrix P.

$$P=\boldsymbol{q}\boldsymbol{q}^{T}+(q_0 I-\boldsymbol{q})^2=\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \tag{18}$$

where

$$\boldsymbol{q}=\begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{19}$$

4.2 On-orbit calibration for star sensor with EKF

The lens distortion can be calibrated with target stars' unit direction-vectors in the star sensor coordinate system and the inertial coordinate system. The theoretical basis of on-orbit calibration is the principle of angular distances' equality, that is

$$\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{W}_j=\boldsymbol{V}_i^{\mathrm{T}}\boldsymbol{V}_j \tag{20}$$

The calibration of lens distortion is actually the estimation of distortion coefficients a_1 to a_7. It can be easily observed that the estimation system is a nonlinear system. Therefore the Extended Kalman Filter (EKF) is used to fulfill this task. The system equations is described as

$$\begin{cases} \xi(k+1)=f(k,\xi(k))+w(k) \\ y(k)=h(k,\xi(k))+v(k) \end{cases} \tag{21}$$

where $\xi(k)$ is the deviation of the real distortion coefficients a_i and the estimated distortion coefficients $\hat{a}_i$. And $y(k)$ is the deviation of the real angular distances and the estimated angular distances after k times' calibration. They are described by equation (22) and equation (23).

$$\xi(k)=\begin{bmatrix} \Delta a_1 \\ \vdots \\ \Delta a_7 \end{bmatrix} \tag{22}$$

$$y(k)=\begin{bmatrix} \boldsymbol{W_1}^{\mathrm{T}}\boldsymbol{W_2}-\boldsymbol{V_1}^{\mathrm{T}}\boldsymbol{V_2} \\ \vdots \\ \boldsymbol{W_1}^{\mathrm{T}}\boldsymbol{W_{num}}-\boldsymbol{V_1}^{\mathrm{T}}\boldsymbol{V_{num}} \\ \boldsymbol{W_2}^{\mathrm{T}}\boldsymbol{W_3}-\boldsymbol{V_2}^{\mathrm{T}}\boldsymbol{V_3} \\ \vdots \\ \boldsymbol{W_{num-1}}^{\mathrm{T}}\boldsymbol{W_{num}}-\boldsymbol{V_{num-1}}^{\mathrm{T}}\boldsymbol{V_{num}} \end{bmatrix} \tag{23}$$

where num is the number of imaged target stars used in this calibration.

In the system equations, $w(k)$ is the systematic noise and $v(k)$ is the measured noise. They conform to the following rules.

$$\begin{cases} \mathrm{E}[w(k)]=0, \ \mathrm{E}[w(k)w(k)^{\mathrm{T}}]=Q^{w}(k) \\ \mathrm{E}[v(k)]=0, \ \mathrm{E}[v(k)V(k)^{\mathrm{T}}]=Q^{v}(k) \\ \mathrm{E}[w(k)v(k)^{\mathrm{T}}]=0 \end{cases} \tag{24}$$

The state variable can be written as

$$\hat{\xi}(k+1)=f(k,\hat{\xi}(k))+N(k)[y(k)-h(k,\hat{\xi}(k))] \tag{25}$$

And the process of measurement update and time update is

$$\begin{aligned} &N(k)=F(k,\hat{\xi}(k))P(k)H^{\mathrm{T}}(k,\hat{\xi}(k))* \\ &[H(k,\hat{\xi}(k))P(k)H^{\mathrm{T}}(k,\hat{\xi}(k))+Q^{v}(k)]^{-1}, \\ &P(k+1)=F(k,\hat{\xi}(k))P(k)F^{\mathrm{T}}(k,\hat{\xi}(k))+Q^{w}(k)- \\ &N(k)[Q^{v}(k)+H(k,\hat{\xi}(k))P(k)H^{\mathrm{T}}(k,\hat{\xi}(k))]N^{\mathrm{T}}(k) \end{aligned} \tag{26}$$

where $F(k,\hat{\xi})$ and $H(k,\hat{\xi})$ are respectively the Jacobi matrix of $f(k,\hat{\xi})$ and $h(k,\hat{\xi})$.

$$F(k,\hat{\xi})=\frac{\partial}{\partial\xi}f(k,\hat{\xi})\bigg|_{\xi=\hat{\xi}}=I \tag{27}$$

$$H(k,\hat{\xi})=\frac{\partial}{\partial\xi}h(k,\hat{\xi})\bigg|_{\xi=\hat{\xi}}= \begin{bmatrix} \frac{\partial}{\partial a_1}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{W}_2 & \cdots & \frac{\partial}{\partial a_7}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{W}_2 \\ \vdots & \cdots & \vdots \\ \frac{\partial}{\partial a_1}\boldsymbol{W}_{num-1}^{\mathrm{T}}\boldsymbol{W}_{num} & \cdots & \frac{\partial}{\partial a_7}\boldsymbol{W}_{num-1}^{\mathrm{T}}\boldsymbol{W}_{num} \end{bmatrix} \tag{28}$$

In the Jacobi matrixes above I is the unit matrix. The derivatives of angular distances for the distortion coefficients can be expressed as

$$\begin{aligned} &\frac{\partial}{\partial a}\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{W}_j=(x_i x_j+y_i y_j)\sqrt{\frac{1}{(x_i^2+y_i^2)(x_j^2+y_j^2)}} \\ &(\frac{\partial\alpha_i}{\partial a}\cos\alpha_i\sin\alpha_j+\frac{\partial\alpha_j}{\partial a}\sin\alpha_i\cos\alpha_j)- \\ &\frac{\partial\alpha_i}{\partial a}\sin\alpha_i\cos\alpha_j-\frac{\partial\alpha_j}{\partial a}\cos\alpha_i\sin\alpha_j \end{aligned} \tag{29}$$

These equations above are the process of EKF. The estimation of state variable can be obtained with the measurements when the initial value of $P(0)$ and $\hat{\xi}(0)$ are given. Fig. 4 illustrates this process perspicuously.

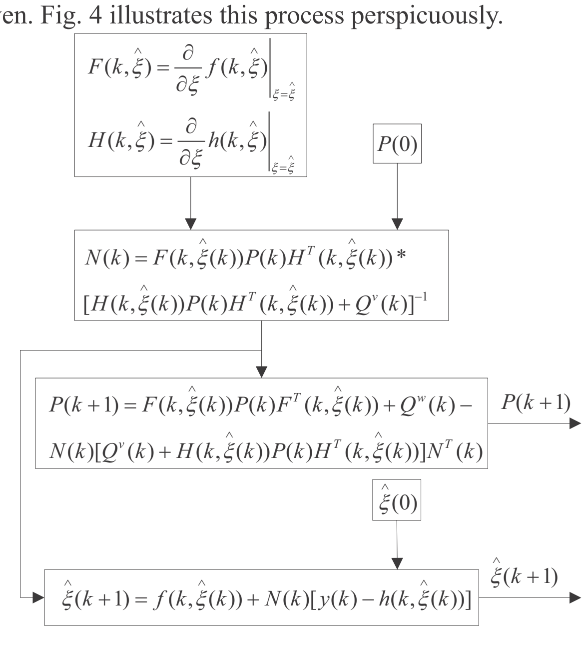

Fig. 4: Flow chart of Extended Kalman Filter

5 Simulation and analysis

The temperature of star sensor's working environment needs to be controlled, and the Peltier devices [8] can keep it below $0^{o}C$. The variation range of target stars' half-field angle in the simulation is 3^{o} to 9^{o}. And the variation range of temperature is $0^{o}C$ to $-50^{o}C$.

Suppose the simulated satellite is a geosynchronous satellite, then its attitude angle rate w is

$$w = \frac{360^{o}}{24*3600s} \approx 0.004^{o}/s \quad (30)$$

The simulated satellite's yaw angle ψ, pitch angle θ and roll angle φ are defined as

$$\psi = -\tan^{-1}(\frac{p_{21}}{p_{22}}),\ \theta = -\tan^{-1}(\frac{p_{13}}{p_{33}}),\ \varphi = \sin^{-1}(p_{23}) \quad (31)$$

The three initial attitude angles are all set to be 10^{o}. In the movement of satellite, the yaw angle and the roll angle don't change while the pitch angle changed from 10^{o} to 16^{o} with the angular rate w. The movement of target stars relative to the star sensor is equivalent to the movement of satellite relative to the inertial space. Thus the angular rate of incident lights' unit direction-vector in the star sensor coordinate system is w too.

The smaller star sensor's time of target-capture is, the more precise its attitude determination will be. However, its time of target-capture is confined by the process of imaging and exposure. Suppose the star sensor's time of target-capture Δt is 0.5s, then the angular deviation of incident lights' unit direction-vector between two adjacent imaging data is

$$\Delta\alpha = w*\Delta t \approx 0.002^{o} \quad (32)$$

That is to say an individual illuminant has an angular variation $\Delta\alpha$ between two adjacent imaging data. The imaging of three target stars has been simulated by ZEMAX with the previous lens model. And there are 33 groups of imaging data used in the on-orbit calibration. The results of calibration are presented as follows.

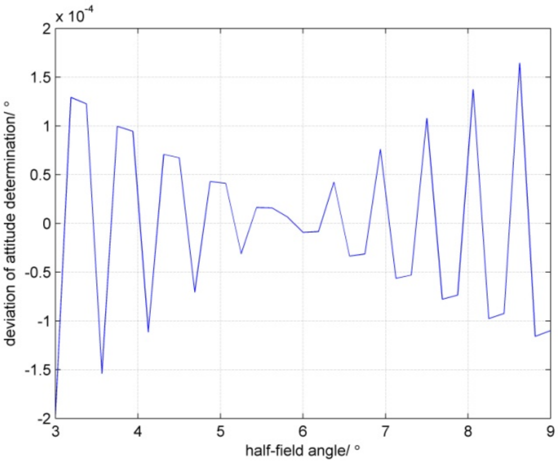

Fig. 5: Deviation of star sensor after calibration

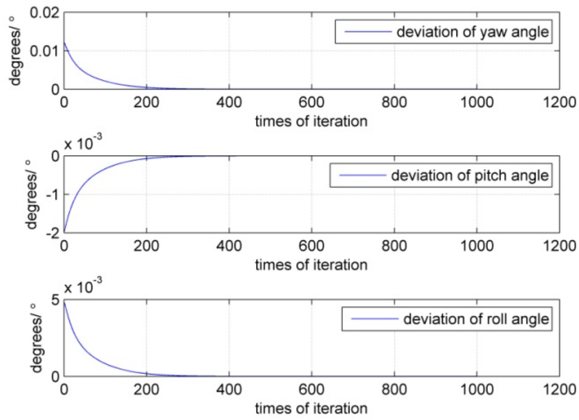

Fig. 6: Deviation of estimated attitude angle

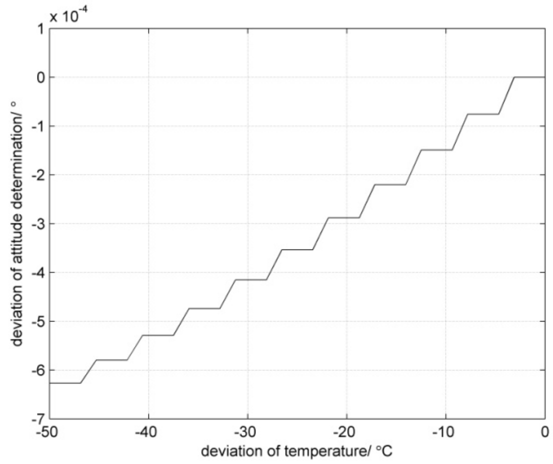

Fig. 7: Deviation of star sensor caused by thermal change

The results of calibration reveal that star sensor' precision of attitude determination varies following the change of temperature and the target star's field angle. The ultimate precision calculated by the algorithm of QUEST is $0.279"$. And the respective precisions of yaw angle, pitch angle and roll angle are $0.208"$, $0.186"$, and $0.197"$. It's easy to see from Fig. 7 that the deviation of attitude determination caused by lens distortion is above $1"$ when the temperature of working environment deviates $20^{o}C$ or more from the design temperature of star sensor lens. This fact demonstrates that the variation of temperature has a big influence on star sensor's attitude determination. And it's necessary to calibrate the lens distortion caused by the variation of temperature to enhance the precision of star sensor.

6 Conclusions

This paper analyzes the defects of the conventional lens distortion model when it's applied to star sensor, that are it doesn't tolerate the variation of temperature and it's based on the over-simplified pin-hole camera model. Then, a star sensor lens model with wide field and large aperture is built with the help of ZEMAX. The new lens distortion model is created by simulating and analyzing the imaging data of this lens model. The new distortion model overcomes the shortcomings of the conventional model successfully. And ultimately an on-orbit EKF algorithm is designed to calibrate star sensor's lens distortion. The results of simulation reveal that the new lens distortion model is effective to eliminate the adverse effect of lens distortion on star sensors. Moreover, the precision of attitude determination by star sensor has been improved markedly.

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