# The Supplement for "The fast heuristic algorithms and post-processing techniques to design large and low-cost communication networks"

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This supplement contains the rigorous proof for the approximation guarantee of our MSTG algorithm, and the detailed computational results in the largest existing benchmark instances and some newly generated instances that are even larger.

## I. The approximation guarantee of the MSTG algorithm

We prove that there is no approximation guarantee for the MSTG algorithm as follows.

**Theorem 1.** *There is no approximation guarantee for the MSTG algorithm.*

*Proof.* Consider the example instance in Figure 1. The MST of this graph is the tree constructed by all the black edges. Since the node weight of each red vertex is $\infty$, no edge can be removed by GPrA from this MST. Thus, the solution of the MSTG algorithm is this MST. Use $T_{MSTG}$ to signify this solution, and $c(T_{MSTG}) = mny$. Use $T_{opt}$ to signify the optimal solution. Suppose $\alpha(m-1) < mn$, then $T_{opt}$ is the tree constructed by all the red edges. Therefore, $c(T_{opt}) = (m-1)x$. We have the approximation ratio of the MSTG algorithm as follows,

$$\lim_{n\to\infty} ratio = \frac{mny}{(m-1)x} > \frac{mn}{\alpha(m-1)} = \infty \qquad (1)$$

Thus, there is no approximation guarantee for the MSTG algorithm. Note that, it is still true when we replace GPrA with P3 in the MSTG algorithm. □

## II. Two tables

We present the detailed computational results of our heuristic algorithms and the state-of-the-art ones in the following two tables, where $|V|$ is the number of vertices, $|E|$ is the number of edges, Previous Best is the previous best-known solution obtained by DA_BB [1] (a new best-known solution is later obtained in our paper), Solu. is the solution obtained by each algorithm, Time is the running time of each algorithm, P3_Solu. is the post-processed solution for each algorithm (bold font is used to highlight the improved solutions), P3_Time is the running time of P3, and GW_LB is the lower bound calculated using Equation (5) in our paper.

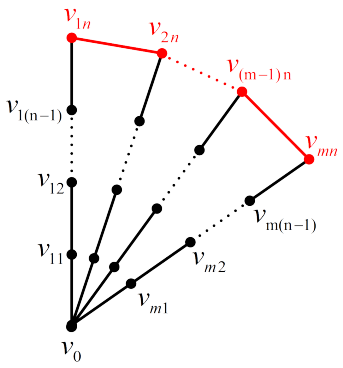


Figure 1: An instance for PCSTP. The node weight of each red vertex $v_{1n}$ to $v_{mn}$ is $\infty$, while that of each black vertex is 0. The cost of each red edge $(v_{1n}, v_{2n})$ to $(v_{(m-1)n}, v_{mn})$ is $x$, while that of each black edge is $y$. $y < x < \alpha y$ and $\alpha > 1$.

## References

[1] M. Leitner, I. Ljubic, M. Luipersbeck, and M. Sinnl, "A dual-ascent-based branch-and-bound framework for the prize-collecting Steiner tree and related problems," tech. rep., Optimization Online, 2016.

[2] C. Hegde, P. Indyk, and L. Schmidt, "A fast, adaptive variant of the Goemans-Williamson scheme for the prize-collecting Steiner tree problem," in *Workshop of the 11th DIMACS Implementation Challenge*, 2014.

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Table I: The computational trials in the existing Hand instances

| Instance | $\|V\|$ | $\|E\|$ | Previous Best (DA_BB [1]) | FGW [2] (OPT1) | | FGW [2] (OPT2) | | FGW′ | | | | MSTG | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Solu. | Time | Solu. | Time | Solu. | Time | P3_Solu. | P3_Time | Solu. | Time | P3_Solu. | P3_Time |
| handbd01 | 169800 | 338551 | 728.9636 | 729.7379 | 0.9242s | 729.1135 | 36.6108s | 729.1135 | 0.9428s | **729.1036** | 0.2458s | 730.1109 | 0.3289s | **729.3935** | 0.2407s |
| handbd02 | 169800 | 338551 | 296.4965 | 297.6809 | 0.2270s | 297.6115 | 23.4267s | 297.6115 | 0.2223s | **297.5754** | 0.3114s | 2227.5098 | 0.3334s | **2143.6177** | 0.2456s |
| handbd03 | 169800 | 338551 | 135.0706 | 135.122 | 1.0693s | 135.0822 | 40.5322s | 135.0706 | 1.1363s | 135.0706 | 0.1232s | 135.0973 | 0.3219s | **135.0790** | 0.2395s |
| handbd04 | 169800 | 338551 | 1813.9592 | 1831.115 | 0.4892s | 1831.115 | 31.7381s | 1831.115 | 0.4867s | **1830.7270** | 0.2896s | 3019.4724 | 0.3193s | **2957.8595** | 0.2418s |
| handbd05 | 169800 | 338551 | 105.4747 | 105.5042 | 1.0446s | 105.5003 | 38.8565s | 105.4747 | 1.0675s | 105.4747 | 0.1234s | 105.4992 | 0.3179s | **105.4946** | 0.2491s |
| handbd06 | 169800 | 338551 | 1528.7654 | 1541.5242 | 0.4737s | 1541.225 | 32.4908s | 1541.225 | 0.5016s | **1540.7989** | 0.2948s | 3250.2024 | 0.3268s | **3129.1386** | 0.2427s |
| handbd07 | 169800 | 338551 | 77.862 | 77.8965 | 1.1763s | 77.8959 | 43.5286s | 77.8625 | 1.1979s | 77.8625 | 0.1237s | 77.8879 | 0.3281s | **77.8865** | 0.2422s |
| handbd08 | 169800 | 338551 | 1368.1668 | 1378.5156 | 0.4604s | 1378.2537 | 29.9428s | 1378.2537 | 0.4403s | **1378.1311** | 0.2944s | 3414.5311 | 0.3257s | **3276.3112** | 0.2529s |
| handbd09 | 169800 | 338551 | 62.7172 | 62.7625 | 1.0605s | 62.7337 | 37.9934s | 62.7175 | 1.0256s | 62.7175 | 0.1262s | 62.7403 | 0.3172s | **62.7337** | 0.2416s |
| handbd10 | 169800 | 338551 | 1137.4297 | 1145.8061 | 0.3885s | 1143.9666 | 28.8286s | 1143.9666 | 0.3896s | **1143.8244** | 0.3190s | 3555.8536 | 0.3243s | **3258.9696** | 0.2511s |
| handbd11 | 169800 | 338551 | 46.7725 | 46.7853 | 1.2141s | 46.7853 | 41.6638s | 46.7725 | 1.2041s | 46.7725 | 0.1234s | 46.779 | 0.3205s | 46.779 | 0.1217s |
| handbd12 | 169800 | 338551 | 321.2047 | 321.4016 | 0.4624s | 321.4016 | 32.0761s | 321.4016 | 0.4804s | **321.3986** | 0.2992s | 2663.0152 | 0.3252s | **2656.4811** | 1.4860s |
| handbd13 | 169800 | 338551 | 13.1889 | 13.2327 | 1.0310s | 13.2148 | 39.3235s | 13.2111 | 1.0106s | 13.2111 | 0.1305s | 13.2241 | 0.3210s | **13.2210** | 0.2395s |
| handbd14 | 169800 | 338551 | 4379.1042 | 4433.4783 | 0.1106s | 4379.1857 | 18.4805s | 4379.1857 | 0.1116s | 4379.1857 | 0.1246s | 4428.7641 | 0.3179s | **4379.3222** | 0.2420s |
| handbi01 | 158400 | 315808 | 1358.5634 | 1361.1002 | 0.9966s | 1359.2352 | 36.6247s | 1358.814 | 0.9456s | **1358.8079** | 0.2394s | 1360.1763 | 0.2978s | **1359.9244** | 0.2380s |
| handbi02 | 158400 | 315808 | 531.8109 | 533.3078 | 0.2013s | 533.3078 | 21.9973s | 533.3078 | 0.1973s | 533.3078 | 0.1428s | 4128.3691 | 0.3059s | **4004.8070** | 0.2356s |
| handbi03 | 158400 | 315808 | 243.1342 | 243.2314 | 1.0698s | 243.2314 | 39.8263s | 243.1342 | 1.0701s | 243.1342 | 0.1207s | 243.2072 | 0.2997s | **243.1851** | 0.2325s |
| handbi04 | 158400 | 315808 | 3202.1857 | 3241.4074 | 0.5057s | 3240.0417 | 30.2661s | 3240.0417 | 0.4610s | **3239.7863** | 0.2695s | 5498.7375 | 0.3010s | **5436.0946** | 0.2313s |
| handbi05 | 158400 | 315808 | 184.4673 | 184.5456 | 1.0612s | 184.4876 | 36.9983s | 184.4673 | 1.0620s | 184.4673 | 0.1183s | 184.4973 | 0.2985s | **184.4782** | 0.2300s |
| handbi06 | 158400 | 315808 | 2921.5447 | 2951.3871 | 0.3950s | 2951.0213 | 27.2265s | 2951.0213 | 0.3797s | **2950.7282** | 0.2711s | 5817.5678 | 0.3014s | **5744.2290** | 0.2322s |
| handbi07 | 158400 | 315808 | 150.9743 | 151.0662 | 0.9616s | 151.0132 | 36.0864s | 150.977 | 0.9531s | 150.977 | 0.1186s | 151.0203 | 0.2978s | **151.0184** | 0.2365s |
| handbi08 | 158400 | 315808 | 2270.2846 | 2285.0306 | 0.3343s | 2284.5631 | 25.4304s | 2284.5631 | 0.3272s | **2284.3594** | 0.2735s | 6055.4094 | 0.3027s | **5888.0953** | 0.2392s |
| handbi09 | 158400 | 315808 | 107.7688 | 107.8008 | 0.9800s | 107.7967 | 38.0417s | 107.77 | 1.0044s | 107.77 | 0.1191s | 107.7905 | 0.3002s | **107.7860** | 0.2320s |
| handbi10 | 158400 | 315808 | 1874.293 | 1881.7441 | 0.2690s | 1880.5621 | 24.4678s | 1880.5621 | 0.2744s | **1880.5208** | 0.2796s | 6375.8028 | 0.3024s | **4545.9725** | 0.2550s |
| handbi11 | 158400 | 315808 | 68.9447 | 68.959 | 1.1458s | 68.959 | 42.5903s | 68.9447 | 1.1929s | 68.9447 | 0.1184s | 68.9493 | 0.3001s | 68.9493 | 0.1186s |
| handbi12 | 158400 | 315808 | 138.257 | 138.3566 | 0.1713s | 138.3002 | 20.1980s | 138.3002 | 0.1714s | 138.3002 | 0.1410s | 1202.2414 | 0.3036s | 1202.2414 | 0.6563s |
| handbi13 | 158400 | 315808 | 4.2745 | 4.3333 | 0.9399s | 4.3139 | 38.3010s | 4.3139 | 0.9455s | 4.3139 | 0.1279s | 4.4289 | 0.3015s | 4.4289 | 0.1161s |
| handbi14 | 158400 | 315808 | 7881.7687 | 7881.9025 | 0.1039s | 7881.9025 | 18.0950s | 7881.9025 | 0.1036s | 7881.9025 | 0.1408s | 8029.2983 | 0.3017s | **7881.9582** | 0.2852s |
| handsd01 | 42500 | 84475 | 171.6368 | 171.9393 | 0.1576s | 171.7123 | 6.4092s | 171.6368 | 0.1562s | 171.6368 | 0.0298s | 171.7438 | 0.0791s | **171.6638** | 0.0548s |
| handsd02 | 42500 | 84475 | 159.7514 | 161.2727 | 0.0557s | 161.1133 | 5.2666s | 161.1133 | 0.0490s | 161.1133 | 0.0327s | 580.9579 | 0.0779s | **576.5816** | 0.0547s |
| handsd03 | 42500 | 84475 | 31.3063 | 31.3319 | 0.1625s | 31.328 | 6.4554s | 31.3063 | 0.1630s | 31.3063 | 0.0300s | 31.3215 | 0.0785s | **31.3209** | 0.0560s |
| handsd04 | 42500 | 84475 | 491.7332 | 496.3119 | 0.0856s | 496.3119 | 5.8644s | 496.3119 | 0.0726s | **496.2860** | 0.0629s | 784.0524 | 0.0797s | **780.1176** | 0.0558s |
| handsd05 | 42500 | 84475 | 21.9376 | 21.952 | 0.1654s | 21.952 | 6.9209s | 21.9376 | 0.1709s | 21.9376 | 0.0283s | 21.951 | 0.0775s | **21.9434** | 0.0539s |
| handsd06 | 42500 | 84475 | 279.9031 | 281.026 | 0.0943s | 280.9208 | 5.9990s | 280.9208 | 0.0725s | **280.9206** | 0.0647s | 850.0455 | 0.0785s | **813.0007** | 0.0574s |
| handsd07 | 42500 | 84475 | 11.8041 | 11.8082 | 0.1974s | 11.8082 | 7.5797s | 11.8041 | 0.2006s | 11.8041 | 0.0286s | 11.8065 | 0.0769s | 11.8065 | 0.0134s |
| handsd08 | 42500 | 84475 | 143.2377 | 143.307 | 0.0414s | 143.307 | 4.6650s | 143.307 | 0.0379s | 143.307 | 0.0357s | 913.7864 | 0.0795s | **611.2809** | 0.0648s |
| handsd09 | 42500 | 84475 | 3.8187 | 3.8271 | 0.1907s | 3.8205 | 7.7641s | 3.8187 | 0.1995s | 3.8187 | 0.0278s | 3.8192 | 0.0780s | 3.8192 | 0.0272s |
| handsd10 | 42500 | 84475 | 1034.7674 | 1045.89 | 0.0260s | 1034.7674 | 4.2425s | 1034.7674 | 0.0257s | 1034.7674 | 0.0290s | 1044.0143 | 0.0774s | **1034.8327** | 0.0552s |
| handsi01 | 39600 | 78704 | 295.4536 | 295.8089 | 0.1431s | 295.6686 | 5.4401s | 295.4953 | 0.1457s | **295.4703** | 0.0428s | 295.6767 | 0.0710s | **295.4893** | 0.0429s |
| handsi02 | 39600 | 78704 | 125.4294 | 125.6777 | 0.0316s | 125.5538 | 3.8797s | 125.5538 | 0.0328s | 125.5538 | 0.0279s | 971.3768 | 0.0720s | **824.3563** | 0.0543s |
| handsi03 | 39600 | 78704 | 56.1494 | 56.242 | 0.2415s | 56.2419 | 7.2809s | 56.1494 | 0.2094s | 56.1494 | 0.0224s | 56.2308 | 0.0704s | **56.2304** | 0.0418s |
| handsi04 | 39600 | 78704 | 722.5082 | 729.5874 | 0.0490s | 729.0027 | 4.2719s | 729.0027 | 0.0514s | **728.9148** | 0.0545s | 1353.3911 | 0.0706s | **1344.8666** | 0.0433s |
| handsi05 | 39600 | 78704 | 35.0435 | 35.0598 | 0.1908s | 35.0598 | 6.3690s | 35.0435 | 0.1856s | 35.0435 | 0.0213s | 35.0494 | 0.0702s | 35.0494 | 0.0213s |
| handsi06 | 39600 | 78704 | 452.9536 | 455.8024 | 0.0456s | 454.4759 | 4.2280s | 454.4759 | 0.0480s | 454.4759 | 0.0284s | 1462.0777 | 0.0708s | **1396.0978** | 0.0444s |
| handsi07 | 39600 | 78704 | 18.4101 | 18.4287 | 0.1663s | 18.4287 | 5.9944s | 18.4101 | 0.1739s | 18.4101 | 0.0215s | 18.4176 | 0.0703s | **18.4132** | 0.0420s |
| handsi08 | 39600 | 78704 | 229.5299 | 229.7778 | 0.0287s | 229.6325 | 3.6339s | 229.6325 | 0.0297s | 229.6325 | 0.0253s | 1603.4522 | 0.0724s | **420.9894** | 0.0607s |
| handsi09 | 39600 | 78704 | 5.9622 | 5.9817 | 0.1627s | 5.9627 | 5.8678s | 5.9622 | 0.1598s | 5.9622 | 0.0218s | 5.9819 | 0.0703s | 5.9819 | 0.0216s |
| handsi10 | 39600 | 78704 | 1803.6975 | 1805.629 | 0.0205s | 1805.629 | 3.3605s | 1805.629 | 0.0215s | **1805.4473** | 0.0500s | 1833.9684 | 0.0777s | **1831.6729** | 0.0443s |

Table II: The computational trials in the newly generated M instances

| Instance | $\|V\|$ | $\|E\|$ | FGW [2] (OPT1) | | FGW [2] (OPT2) | | FGW$'$ | | | | | MSTG | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Solu. | Time | Solu. | Time | Solu. | Time | GW_LB | P3_Solu. | P3_Time | Solu. | Time | P3_Solu. | P3_Time |
| M1A | 1000000 | 10000000 | 5481.6409 | 41.1174s | 5481.632 | 278.9158s | 5481.632 | 41.1802s | 5460.6945 | **5481.2040** | 2.7353s | 5493.5729 | 5.9965s | **5490.3174** | 3.0465s |
| M2A | 1000000 | 10000000 | 5488.2152 | 35.6940s | 5488.2152 | 239.3660s | 5488.2152 | 36.0353s | 5466.5515 | **5487.8637** | 2.8222s | 5499.5815 | 6.4923s | **5496.7685** | 3.1177s |
| M3A | 1000000 | 10000000 | 5482.2617 | 45.0729s | 5482.244 | 304.4540s | 5482.244 | 45.3488s | 5461.1391 | **5481.8221** | 2.8175s | 5493.1467 | 6.7244s | **5490.0110** | 2.9944s |
| M4A | 1000000 | 10000000 | 5486.2941 | 43.5985s | 5486.2692 | 289.2165s | 5486.2692 | 44.0277s | 5464.7241 | **5485.9517** | 2.9113s | 5497.0325 | 6.6048s | **5494.0865** | 3.0571s |
| M5A | 1000000 | 10000000 | 5486.4648 | 46.7103s | 5486.4498 | 308.3590s | 5486.4498 | 46.8574s | 5465.042 | **5486.0817** | 3.1080s | 5497.1304 | 6.6118s | **5494.4981** | 3.1362s |
| M6A | 1000000 | 10000000 | 5483.5598 | 41.6654s | 5483.5598 | 277.2854s | 5483.5598 | 41.9960s | 5461.4035 | **5483.2009** | 3.0440s | 5494.292 | 6.5396s | **5491.6260** | 3.2399s |
| M7A | 1000000 | 10000000 | 5483.2707 | 37.6512s | 5483.2707 | 253.3655s | 5483.2707 | 38.8013s | 5461.3939 | **5482.9075** | 3.0819s | 5493.7591 | 6.7919s | **5490.8884** | 3.2503s |
| M8A | 1000000 | 10000000 | 5488.9262 | 44.0611s | 5488.9202 | 295.2093s | 5488.9202 | 44.7165s | 5466.709 | **5488.5406** | 3.2238s | 5500.0362 | 6.8583s | **5497.1859** | 3.3526s |
| M9A | 1000000 | 10000000 | 5489.351 | 43.9632s | 5489.3345 | 288.1966s | 5489.3345 | 45.0848s | 5467.2073 | **5488.9733** | 3.1806s | 5500.1045 | 6.7159s | **5497.4085** | 3.4248s |
| M10A | 1000000 | 10000000 | 5483.7157 | 45.2397s | 5483.7009 | 301.1237s | 5483.7009 | 45.9673s | 5461.8778 | **5483.3028** | 3.2424s | 5493.9648 | 6.0955s | **5491.2744** | 3.4998s |
| M11A | 1000000 | 10000000 | 5484.0646 | 45.7019s | 5484.0537 | 304.1795s | 5484.0537 | 46.7220s | 5462.7545 | **5483.6038** | 3.5856s | 5494.972 | 6.4800s | **5492.1123** | 3.5348s |
| M12A | 1000000 | 10000000 | 5485.794 | 47.3249s | 5485.794 | 317.2440s | 5485.794 | 48.0205s | 5464.3387 | **5485.3847** | 3.4342s | 5497.0247 | 6.4409s | **5493.6634** | 3.7350s |
| M13A | 1000000 | 10000000 | 5481.4588 | 47.3713s | 5481.4431 | 317.3814s | 5481.4431 | 48.4331s | 5459.5761 | **5481.0153** | 3.5405s | 5492.5614 | 6.6036s | **5489.9386** | 3.9404s |
| M14A | 1000000 | 10000000 | 5481.7737 | 45.5650s | 5481.7078 | 304.6331s | 5481.7078 | 47.8858s | 5460.3079 | **5481.4068** | 3.4022s | 5492.144 | 6.5634s | **5489.0575** | 3.6227s |
| M15A | 1000000 | 10000000 | 5484.197 | 47.4944s | 5484.1881 | 318.1793s | 5484.1881 | 47.7006s | 5462.8524 | **5483.7696** | 3.5510s | 5494.7691 | 6.2155s | **5492.5581** | 3.8557s |
| M16A | 1000000 | 10000000 | 5487.2783 | 47.5215s | 5487.2783 | 315.4937s | 5487.2783 | 48.1742s | 5465.2008 | **5486.8723** | 3.6924s | 5497.5028 | 6.4870s | **5494.3715** | 3.8062s |
| M17A | 1000000 | 10000000 | 5483.7691 | 42.6993s | 5483.7476 | 281.9618s | 5483.7476 | 42.9462s | 5461.9474 | **5483.3123** | 3.6478s | 5494.721 | 6.2138s | **5492.2295** | 3.8779s |
| M18A | 1000000 | 10000000 | 5486.6205 | 43.7577s | 5486.6117 | 288.7708s | 5486.6117 | 44.7528s | 5465.2081 | **5486.3287** | 3.6500s | 5498.0079 | 6.7424s | **5494.6275** | 3.6404s |
| M19A | 1000000 | 10000000 | 5483.9776 | 45.8445s | 5483.9695 | 307.5425s | 5483.9695 | 46.4310s | 5462.1784 | **5483.5960** | 3.4813s | 5495.2559 | 6.8354s | **5492.3760** | 3.6918s |
| M20A | 1000000 | 10000000 | 5487.5845 | 46.2689s | 5487.5651 | 308.2115s | 5487.5651 | 46.8482s | 5465.6964 | **5487.1075** | 3.5149s | 5498.6258 | 6.7236s | **5495.8071** | 3.8611s |
| M1B | 1000000 | 10000000 | 66.2021 | 32.3069s | 66.1832 | 247.5397s | 66.1832 | 32.4441s | 53.3145 | **66.0466** | 2.8262s | 92.4365 | 6.3559s | **79.3268** | 3.1068s |
| M2B | 1000000 | 10000000 | 66.0324 | 19.8363s | 66.0174 | 168.0830s | 66.0174 | 19.9129s | 52.5769 | **65.8882** | 2.7637s | 94.1092 | 6.7074s | **80.0447** | 3.1112s |
| M3B | 1000000 | 10000000 | 65.7169 | 30.4805s | 65.7068 | 236.2986s | 65.7068 | 30.9065s | 52.816 | **65.5986** | 2.8050s | 92.9147 | 7.0690s | **82.2103** | 3.2348s |
| M4B | 1000000 | 10000000 | 65.4162 | 26.0066s | 65.394 | 202.1854s | 65.394 | 26.2628s | 52.3969 | **65.2567** | 2.7958s | 93.8008 | 6.8425s | **81.0745** | 3.1842s |
| M5B | 1000000 | 10000000 | 65.6691 | 28.3675s | 65.6691 | 219.7546s | 65.6691 | 28.6480s | 52.3289 | **65.5881** | 2.7750s | 93.0645 | 6.3484s | **80.7955** | 3.0979s |
| M6B | 1000000 | 10000000 | 65.847 | 31.1829s | 65.8336 | 237.4612s | 65.8336 | 31.3760s | 52.6452 | **65.7684** | 2.8360s | 92.6384 | 6.9973s | **79.4922** | 3.1287s |
| M7B | 1000000 | 10000000 | 65.5548 | 29.2183s | 65.5447 | 226.8594s | 65.5447 | 29.4760s | 52.5285 | **65.3970** | 2.7788s | 92.0847 | 7.0038s | **79.7433** | 3.1347s |
| M8B | 1000000 | 10000000 | 66.1979 | 28.1588s | 66.1815 | 217.9342s | 66.1815 | 28.1823s | 52.9914 | **66.0255** | 2.8414s | 92.4475 | 7.0138s | **78.1951** | 2.9598s |
| M9B | 1000000 | 10000000 | 65.429 | 31.5468s | 65.4242 | 241.1995s | 65.4242 | 32.2525s | 52.2148 | **65.3305** | 2.7388s | 92.4493 | 7.2335s | **79.6682** | 3.2323s |
| M10B | 1000000 | 10000000 | 65.5545 | 32.6414s | 65.5444 | 246.0274s | 65.5444 | 31.9424s | 52.5864 | **65.4162** | 2.8588s | 93.9597 | 7.0986s | **82.2695** | 3.2284s |
| M11B | 1000000 | 10000000 | 65.5385 | 26.3979s | 65.5277 | 207.4508s | 65.5277 | 26.1448s | 52.5764 | **65.3598** | 2.8288s | 92.2234 | 7.1162s | **78.3228** | 3.0511s |
| M12B | 1000000 | 10000000 | 66.1164 | 30.8797s | 66.1085 | 237.9132s | 66.1085 | 30.8732s | 52.8355 | **65.9770** | 2.8050s | 93.8467 | 7.0847s | **78.6274** | 3.0463s |
| M13B | 1000000 | 10000000 | 65.7901 | 30.6637s | 65.7772 | 232.8243s | 65.7772 | 30.5422s | 52.5653 | **65.6787** | 2.7308s | 93.9122 | 7.1522s | **81.4700** | 3.1073s |
| M14B | 1000000 | 10000000 | 65.6827 | 30.3201s | 65.6772 | 230.7312s | 65.6772 | 29.9101s | 52.3993 | **65.6211** | 2.8106s | 93.316 | 6.4010s | **79.6478** | 2.9961s |
| M15B | 1000000 | 10000000 | 65.4326 | 29.3813s | 65.4123 | 227.3946s | 65.4123 | 29.8681s | 52.5718 | **65.3125** | 2.7157s | 92.1817 | 7.2176s | **79.5161** | 3.0208s |
| M16B | 1000000 | 10000000 | 66.0073 | 30.9838s | 65.9965 | 237.9746s | 65.9965 | 30.9989s | 52.4984 | **65.9059** | 2.7464s | 92.6824 | 7.1303s | **81.2880** | 3.1640s |
| M17B | 1000000 | 10000000 | 65.7726 | 28.6853s | 65.7602 | 223.1243s | 65.7602 | 28.6653s | 52.582 | **65.6592** | 2.7328s | 94.4974 | 7.0025s | **81.5226** | 3.1198s |
| M18B | 1000000 | 10000000 | 65.3082 | 28.4717s | 65.3016 | 220.4450s | 65.3016 | 28.8249s | 52.3314 | **65.1425** | 2.8311s | 92.8887 | 7.1751s | **81.3532** | 3.1336s |
| M19B | 1000000 | 10000000 | 65.5194 | 32.5167s | 65.5194 | 253.1833s | 65.5194 | 32.9705s | 52.5215 | **65.4110** | 2.7679s | 93.1059 | 6.9005s | **82.7996** | 3.2278s |
| M20B | 1000000 | 10000000 | 66.0938 | 32.8374s | 66.0804 | 255.3113s | 66.0804 | 32.9264s | 52.7985 | **65.9690** | 2.7642s | 92.8594 | 6.6573s | **80.8396** | 3.1067s |